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VOLUME TABLE CONSTRUCTION

By

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(A manual for the collection of volume table data from felled trees and a suggested method of volume table construction)

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A volume table may be defined as a tabular statement of the average volume of trees of specific sizes. Volume tables generally list volume in either board feet, cubic feet, or cords, and may be of a basic type termed local, standard, or form class volume tables. This classification depends on the variables used to express the volume as presented in the table. All volume tables have limitations, not the least among which are the varying degrees of utilization contemplated by the ultimate user of the table.

There has been a strong tendency in recent years towards the use of composite volume tables with correction factors for local variations. These tables have been quite satisfactory when used over large areas, such as state-wide inventories, or when adjusted (usually for form class) to more localized conditions. They are sufficiently accurate for the purposes of most users.

Composite volume tables, without adjustments, are not accurate enough for research purposes. Furthermore, the measurements that are needed in order to determine whether a composite table should be adjusted are quite often as detailed as those needed for constructing a completely new standard volume table for local use. Gevorkiantz and Olsen [1/] indicate that

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1/ Gevorkiantz, S. R. and L. P. Olsen. 1955. Composite Volume Tables for Timber and Their Application in the Lake States. Tech. Bull. No. 1104. U.S.D.A.

approximately 25 sample tree measurements are needed to make the necessary check for composite volume table adjustment when the desired limit of accuracy is ± 6 percent. To obtain an accuracy of ± 3 percent 100 sample tree measurements are needed.

The concept of using volume as a relative comparison is not entirely a sound procedure in research investigations. The researcher's chief concern is in absolute changes brought about by his experimentation. Consequently, a volume table which is accurate before treatment may not be accurate 5 years after treatment in the same stand or plot. The research forester is periodically confronted with problems of volume table selection, adjustment, and construction. Occasionally he has no choice but to construct new volume tables based on a carefully selected sample of trees that will give him the accuracy he needs.

COLLECTING THE DATA

The following instructions are meant to serve as a guide in collecting volume table data. Felled tree measurements provide the most reliable source of volume table data and the instructions given here pertain to this kind of data. The data will serve the following purposes:

1. As a check on existing volume tables now in use in the area where the data are collected.
2. As a basis for the construction of new volume tables for the use of the Research Centers in managing their Experimental forests.
3. To check volume tables from adjacent regions for possible use in the central states region.
4. For possible use in the construction of region-wide standard volume tables.

Characteristics of the Sample. It is very important that the sample trees used for volume table construction are representative of the area to which the table will be applied. Many characteristics must be considered and a good bit of judgment rests on the field crew. A representative sample should depict the essential characteristics that affect volume, namely height, form, and upper log taper. Sample trees for a given species should be well distributed over the area where the volume table is to be applied. Is the volume table constructed for use in a single compartment, sale area, or study area or is it for a larger area such as an entire ranger district or an experimental forest? Within these areas no special sizes or growing conditions should be unduly represented. If sample trees are used from a logging operation they should be representative of the trees that are left. Harvest cuttings are generally a better source of data than are intermediate cuttings. The data recorded on the top portion of the tally sheet will be quite helpful in evaluating the site conditions of the sample tree. Any irregularities or unique conditions pertaining to past use of the area, the purpose of the cutting, defect, etc. should be recorded in the remarks section of the tally sheet.

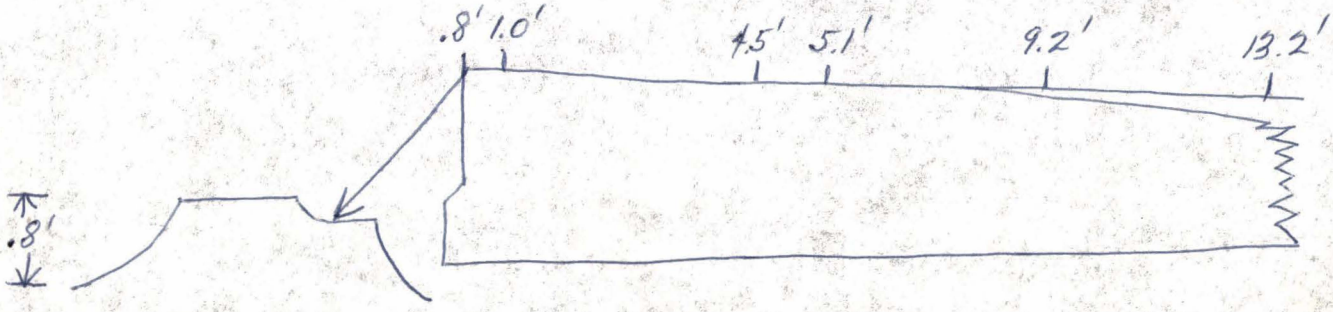
Tools and Equipment.

1. 1 axe suitable for limbing.
2. 1 100-foot steel tape.
3. 1 25-foot cloth tape.
4. 1 percent abney or suitable hypsometer for measuring the heights of standing trees.
5. 1 properly adjusted diameter caliper. If a 4-man crew is used a second caliper can be used to advantage.
6. Several 10d nails and a small hammer.
7. Lumber crayon; yellow or red.
8. Clip board, tally sheets and 4 or 5-H pencils.
9. 2 bark guages.

Measuring and Recording the Data. The tally sheet (Form CS-333) has been designed to facilitate office computations. Those figures recorded in blue are field recordings and the red figures are office computations. The following discussion is a step by step account of how each field recording is made with some suggested procedures for obtaining the measurements.

Record individual species by tree numbers in the order measured. The locality should be identified by some specific feature. The stand size should be classified into one of five classes depending upon which class is predominant. For example, a stand composed of a few scattered sawtimber trees with many pole size trees underneath would be classified as a pole-sized stand. The stand type should also be classified on the basis of the predominant species or group of species.

All measurements should be made on felled trees before they are bucked into logs. The age of the tree is equal to the ring count on the stump plus 1. Before taking diameter and height measurements the field crew should remove any small branches (less than 3") that might interfere with the placement of the steel tape on the upper side of the bole of the tree. Stump height should be measured on the same side of the tree on which the stem diameter points are marked (see diagram below).



INDIVIDUAL TREE TALLY SHEET FOR VOLUME TABLE MEASUREMENTS

Tree species Pin Oak
Tree number 1
State Ill
County [illegible]
Locality [illegible]
Crew [illegible]
Date [illegible]

Topo: Ridge; Slope U M L; Bottomland; Cove;
Stand size: Sawtimber - Large (21" dbh & up)
Medium (13 - 21")
Small (9 - 13")
Pole timber (5 - 9")
Sapling (less than 5")
Stand type: Mixed hardwoods; oak-hickory; oak;
oak-pine; pine; other Pin Oak

D.B.H. o.b. 17.1 inches
Total height [illegible] feet
Height to 4.5 inch top o.b. 72.3 feet
Height for saw logs 47.1 feet (46 ft. merch.)

Form class 82 percent
Age (stump count + 1) 62 years
Crown class - D or CD; I; S; unknown

Height above ground	D_i	2B	d_i	k	n_i	V	v
feet	in.	in.	in.			cu.ft.	cu.ft.
1.0	20.2	1.4	18.8	.93	1	$\sum D_i^2 . n_i$ $\sum d_i^2 . n_i$	
5.1	16.7	1.2	15.5	.94	4	3259.35 3057.76	24.00 21.17
9.2	16.1		15.1		2		
12.2	15.4		14		4		
3	14.4	0.9	14.0	.94	1	(f = 0.0074086)	
21.4	14.1		13.6			$\sum D_i^2$ $\sum d_i^2$	
29.5	13.5	0.8	12.8	.95			
37.7	12.3		11.6			754.52 662.12	33.54 29.43
45.8	11.1	0.7	9.7	.92		(f = 0.044451)	
54.0	8.8		7.8			Last section and branch sect.	
62.1	6.7	0.6	6.1	.92		D_m 2B d_m L k	
70.3						9.1 0.1 8.2 6.1 .91	.13 .64
78.4						Total excluding branchwood	58.37 51.75
86.6						4.6 0.1 4.2 5 .92	.91 .77
94.7						4.3 0.1 4.2 1.1	1.51 1.31
102.9							

Limits of Merchantability:
Sawlogs, Hardwoods 8.5" top o.b.
Softwoods 6.5" top o.b.

Total volume, cubic feet 60.10 53.0

Board foot volumes:	D_u	2B	d_u	L	Vol. Int. 1/4"	Vol. Scribner
	in.	in.	in.	ft.	bd. ft.	bd. ft.
1st log	14.9	0.1	14.0	16	136	123
2nd log	12.5	0.1	12.2	13	94	80
3rd log	9.7	0.1	9.0	17	49	37
4th log						
5th log						
6th log						

Totals: 279 240
Remarks:

Legend: D_i, diameter outside bark (d.o.b.); d_i, diameter inside bark (d.i.b.); D_m, midpoint d.o.b.; d_m, midpoint d.i.b.; D_u, upper end d.o.b.; d_u, upper end d.i.b.; 2B, double bark thickness; k, ratio of d.i.b. to d.o.b.; f, multiplier used in summarization; L, length; V, volume including bark; v, volume excluding bark.

CS-333

With the tape anchored as shown in the diagram the points of measurement as listed on the tally sheet should be marked with lumber crayon. The procedure for taking the measurements is as follows:

Two diameter measurements are taken at right angles to each other at the specified points on the stem and the average recorded to the nearest tenth of an inch. These measurements are made with calipers. Diameter tapes may be used but calipers are preferred in this case due to the difficulty of using a tape on a felled tree. Double bark thickness (2B) is the sum of two measurements, also taken at right angles, and recorded to the nearest tenth of an inch.

Height measurements are taken with a steel tape. The merchantable height measurement for cubic-foot purposes is the height to a minimum 4.5* inch outside bark diameter unless further limited by the tree being defective, excessively forked, or otherwise considered unmerchantable. Height for sawlogs is obtained similarly, but the minimum top diameter is 8.5 inches outside bark. Included in the height for sawlogs is a trimming allowance of .3 feet for every 16-foot log, as reflected in the given heights above ground on the tally sheet. Table 1 shows the relationship between the total length of the merchantable sawlogs and the actual height needed to provide the trimming allowance and 1-foot stump allowance. The merchantable height for sawlogs should be taken at a point that will give a whole foot of length to the last sawlog (46 feet in the example). Total height should be measured on the central stem of the crown.

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* Other minimum top diameters may be desired.

Table 1. Heights Above Ground Needed for Merchantable Sawlog Lengths When Including Trimming Allowance and Stump Allowance

Height above ground feet	Merchantable sawlog length feet	Height above ground feet	Merchantable sawlog length feet
1.0	stump height		
2.0	1	42.8	
3.0		43.8	
4.1		44.8	
5.1		45.8	
6.1	5	46.8	45
7.1		47.9	46
8.1		48.9	47
9.2		49.9	48
10.2		50.9	49
11.2	10	51.9	50
12.2		53.0	51
13.2		54.0	52
14.2		55.0	53
15.3		56.0	54
16.3	15	57.0	55
17.3	16	58.0	
18.3	17	59.1	
19.3	18	60.1	
20.4		61.1	
21.4	20	62.1	60
22.4		63.1	
23.4		64.2	
24.4		65.2	
25.4		66.2	
26.5	25	67.2	65
27.5		68.2	
28.5		69.3	
29.5		70.3	
30.5		71.3	
31.6	30	72.3	70
32.6		73.3	
33.6		74.4	
34.6		75.4	
35.6		76.4	
36.7	35	77.4	75
37.7		78.4	
38.7		79.4	
39.7		80.5	
40.7		81.5	
41.8	40	82.5	80

The crown development should also be noted in order to determine the crown class. Height measurements of a few standing trees near the felled tree might also provide a clue as to the crown position of the felled tree.

The diameter measurements up to and including 17.3 feet above ground (see tally sheet) are end and middle-of-bolt diameters, while above 17.3 feet the measurements are all middle-of-bolt diameters. In the example shown, the point at which the diameter at the middle of the last section is taken was obtained as follows:

$$\frac{\text{Height to 4.5-inch top} - (62.1 + 4.1 \text{ feet including trimming allowance})}{2} + (62.1 + 4.1 \text{ feet including trimming allowance})$$

$$= \frac{72.3 - (62.1 + 4.1)}{2} + (62.1 + 4.1)$$

$$= 3.0 + 66.2 = 69.2 \text{ feet above ground}$$

Branchwood is measured with a cloth tape to the nearest foot of length and to a 4.5-inch diameter outside bark at the upper end of the branch. The diameter measurements for branchwood are taken at the middle of the bolt.

TREE VOLUME COMPUTATIONS FROM FIELD DATA

Cubic feet: The volume below 17.3 feet above ground is computed by Newton's formula applied to the 8-foot bolts in a successive manner. The combined formula, in this case, would be:

$$\text{Volume} = \frac{\left(\frac{\pi}{4} D_0^2 + \frac{4\pi}{4} D_{m_1}^2 + \frac{2\pi}{4} D_1^2 + \frac{4\pi}{4} D_{m_2}^2 + \frac{\pi}{4} D_2^2\right) \frac{L}{12}}{144}$$

where D_o, D_{m_1}, D_1, D_{m_2}, and D_2 are diameters at 1.0, 5.1, 9.2, 13.2, and 17.3 feet above ground and L = 16.3 feet. This formula can be reduced to:

$$V = \left(\frac{\pi}{4} \cdot \frac{L}{12} \cdot \frac{1}{144}\right)\left(\Sigma D_i^2 n_i\right) = f\left(\Sigma D_i^2 n_i\right).$$

where $f = \frac{\pi}{4} \cdot \frac{16.3}{12} \cdot \frac{1}{144} = 0.0074088$

and $\Sigma D_i^2 n_i$ = sum of squares of the diameters each multiplied by the number of times they appear in the original formula, that is, n_i = 1, 4, 2, 4, or 1.

The volume above 17.3 feet is computed by Huber's formula applied to each 8-foot bolt and summed. The expression would be:

$$\text{Volume} = \frac{\Sigma \frac{\pi}{4} \cdot D_i^2 L}{144}$$

where $L = \left(\frac{16.3}{16}\right)(8) = 8.15$

and D_i = the diameters at the middle of the 8-foot bolts.

This can be reduced to:

$$\text{Volume} = \frac{\pi (8.15)}{4(144)} \Sigma D_i^2 = f \Sigma D_i^2$$

where $f = 0.044451$

In the cubic-foot computation, the volumes are computed including bark (using diameters outside bark) and excluding bark (using diameters inside bark). The inside bark diameters, d_i, are obtained by subtraction where the double bark thickness has been recorded in the field. To obtain the remaining d_i's, a value of $k = \frac{d}{D}$ is computed for the points having both D and d. An average value of k is then used in the equation $d_i = kD_i$ to get the inside bark diameters needed.

After determining $\Sigma D_i^2 n_i$, $\Sigma d_i^2 n_i$, ΣD_i^2, and Σd_i^2, the volume, V (including bark) and v (excluding bark) are obtained by multiplying by the appropriate f factor. The volume in the last section and branch sections is determined by multiplying basal area at the middle of the bolt times length.

Board feet: The volume of the merchantable length of sawlogs will be computed by both the International 1/4" log rule and the Scribner log rule.

The International rule can be expressed in a formula as:

Volume = $0.199d^2 - 0.642d$ for a four-foot section

or Volume = $0.796d^2 - 1.375d - 1.230$ for a sixteen-foot log, where d is the diameter inside bark at the upper end of the log.

For logs of lengths between four and sixteen feet, a taper allowance of one-half inch in diameter per four-foot section is made to obtain the diameter at the upper end of the (next) section. If this section is less than four feet the volume is determined for a four-foot section using this upper diameter and multiplied by

$$\frac{\text{actual length of section}}{4}$$

A specially prepared table (table 2) will be utilized to avoid the repetitious solution of these formulae.

Board-foot volumes, in terms of the Scribner log rule, can be obtained by applying the formula:

$$\text{Volume} = .79d^2 - 2d - 4 \text{ for sixteen-foot logs.}$$

For logs shorter than sixteen feet, the volume of a sixteen-foot log is multiplied by

$$\frac{\text{length of the log}}{16}$$

to obtain the partial log volume.

Table 5 was prepared for obtaining the volume of logs of different lengths and diameters.

The Girard Form Class appearing on the tally sheet was computed by the formula

$$\text{G.F.C.} = \frac{\text{dia. inside bark at the top of first 16-foot log}}{\text{diameter breast high outside bark}}$$

CONSTRUCTING THE VOLUME TABLE

Research dealing with the application of statistical procedures in volume table construction has shown that formula methods of construction are superior to the better known graphical and alignment chart methods. One of the best all-around formula from the standpoint of accuracy and simplicity is known as the combined variable formula, in the form

$$\text{Volume} = a + bD^2H$$

where a and b are the regression coefficients and D^2H is the combined variable which is the product of the DBH squared and tree height. The example given here will deal with the use of this equation. The

instructions will be direct and simple, in more or less "cook book" style.

Certain assumptions must be made. Let us assume that we want a cubic foot volume table giving merchantable volume from a 1-foot stump to a 4.5-inch top outside bark. We desire also to include the volume in branchwood. All volumes are to include bark volumes. We want the volume table to show merchantable volume in terms of DBH and <u>total</u> tree height. On the field tally sheet we have six choices of volume, depending on the unit of measure and merchantability standards. They are:

Cubic volume without branchwood, but with bark volume	58.27
Cubic volume without branchwood and without bark volume	51.52
Cubic volume with branchwood and with bark volume	60.70
Cubic volume with branchwood and without bark volume	53.50
Board foot volume, International 1/4 inch rule	279
Board foot volume, Scribner rule	250

From the assumptions given above, the dependent variable Volume (V) is 60.70 cubic feet and the independent variable (D^2H) is $(17.1)^2$ times 86 or 25.147.

Now let us suppose that we have 65 tree measurements for Pin Oak in Southern Illinois and we want to construct a volume table using the combined variable equation. Group the data as shown below*.

Tree No.	Volume (Y)	DBH	DBH^2	Total Height	D^2H (X)	$(X)^2$	(XY)	$(Y)^2$
1	60.70	17.1	292.41	86	25,147	632,371,609	1,526,423	3,684.49
2	29.30	10.6	112.36	65	7,305	53,333,805	213,976	858.49
3	43.85	12.1	146.41	72	10,542	111,133,764	460,158	1,905.32
.	.	.	.	.	.	.	.	.
.	.	.	.	.	.	.	.	.
.	.	.	.	.	.	.	.	.
.	.	.	.	.	.	.	.	.
64	72.75	22.0	484.00	84	40,656	1,652,910,336	2,957,724	5,292.56
65	17.25	8.1	65.61	58	3,805	14,478,025	65,636	297.56
65	2,750.65				713,728	18,980,852,906	65,266,197	251,962.01

*The last three columns are generally accumulated on a desk calculator and only the sums are recorded.

A graphical check can be made at this point in order to obtain a quick estimate of the relationship between volume and D^2H by plotting the individual V's over the corresponding D^2H's. Very seldom will the trend be strong enough to draw an accurate free hand curve. In certain cases, such as young plantations of a single species, or other limited samples of highly uniform conditions, free hand curve fitting may be acceptable.

A method of curve fitting which is free of personal bias and from which the accuracy of the resulting table can be estimated is known as the method of least squares. According to this method, the fit of the regression line is best when the sum of the squared residuals (residual = difference between actual and estimated volume) is a minimum. In the case of the combined variable equation (which is linear), two regression equations must be solved. These equations are:

$$\text{I.} \quad Na + b\Sigma X = \Sigma Y$$

$$\text{II.} \quad a\Sigma X + b\Sigma X^2 = \Sigma XY$$

where N = the number of trees in the sample (65)

Σ signifies the sum of values from tree No. 1 to 65

$X = D^2H$

Y = volume

and

a and b are the regression coefficients which determine the position and slope of the fitted line.

Substituting the totals given in the grouped data the equations can now be written:

$$\text{I.} \quad 65a + 713{,}728b = 2750.65$$

$$\text{II.} \quad 713{,}728a + 16{,}980{,}532{,}906b = 65{,}265{,}197$$

The solution of these equations is not a difficult procedure since there are only two unknowns. It can be accomplished in several ways. In the example given here both sides of equation II were divided by the constant 10,980.45 (715,728/65) and then by subtracting equation I from equation II b can be solved directly. In this case b = 0.003835. The next step is to substitute b in either equation and solve for a. In this case a = 0.208. The regression equation is then

$$\text{Volume} = 0.208 + 0.003835D^2H$$

Volume can now be computed for trees of any combination of diameter and height. For example, a tree with a DBH of 12 inches and a total height of 75 feet has an estimated cubic foot volume of 41.6.

$$\text{Volume} = 0.208 + 0.003835\ (12)^2\ 75$$
$$= 0.208 + 41.418$$
$$= 41.6 \text{ cubic feet}$$

When the intercept constant a is small, as is often the case, the equation can be simplified by merely using the second constant, b. This simplification results in an equation that is essentially the same as the form factor method of calculating volume.

Accuracy of the Volume Table

The accuracy of the regression equation and consequently the accuracy of the volume table estimate can readily be determined from the least squares solution. The first step is to determine the sum of the squared residuals $\Sigma(Y_{\text{actual}} \text{ minus } Y_{\text{estimated}})^2$ as follows:

$$\Sigma (Y_a - Y_c)^2 = \Sigma Y^2 - a\Sigma Y - b\Sigma XY$$

$$= 251,962 - (0.208)(2750.65) - (0.003835)(65,265,197)$$

$$= 1,097.865$$

then, by definition, the standard error of estimate is equal to:

$$\sqrt{\frac{\Sigma (Y_a - Y_c)^2}{N - P}}$$

where N is the number of trees in the sample (65) and P is the number of parameters (in this case two, a and b) characterizing the regression line. Therefore, the standard error of estimate is

$$\sqrt{\frac{1,097,865}{63}} = \pm 4.17 \text{ cubic feet}$$

This is 9.8% of mean volume (2750.65/65 = 42.32). When the individual tree volumes are normally distributed around the mean then it can be stated that the standard error of estimate shows the average range within which the true value will occur about two-thirds of the time. The accuracy of the volume estimate depends on two things: the size of the sample, which can be controlled, and the uniformity of the individual trees in the sample, which cannot be controlled. Suppose that the desired accuracy is $\pm$ 2.0 cubic feet instead of $\pm$ 4.17. How many tree measurements, from the same sample, are needed to obtain this accuracy? A simple rule to follow is: to increase the accuracy x times approximately x^2 times the number of samples are needed. In the present example the desired accuracy is approximately twice the accuracy obtained from the initial sample of 65 trees, hence 4 (2 squared) times as many measurements, or 260, are needed. When certain characteristics of the sample are known beforehand it is often possible to predict the approximate number of sample trees needed to obtain the desired accuracy.